

**Encouraging Risk-Taking in Year 12 Girls Through a Collaborative Approach to Solving Non-Routine Questions in Higher Level Mathematics**

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**Abstract**

Maximising examination readiness for Year 12 students between the internal “trial” examinations and the final state matriculation examinations six school weeks later is always a challenge, especially in the higher levels of Mathematics. Traditionally girls complete a practice paper each week, correct their mistakes, then move onto the next one. They do not necessarily master the techniques they were unable to do or got wrong in the practice papers, leaving them open to the same errors in the final examination. This research project addresses this preparation loophole for a group of ten pre-matriculation girls studying the elite course, Mathematics Extension 2 in New South Wales, Australia. Three elements for girls’ success were identified and implemented as the action of this action research project: first, facilitating Year 12 girls to work collaboratively on mathematical problems to increase confidence; second, working on difficult, non-routine mathematical problems; and third, not providing solutions to mathematical problems. The results are encouraging and suggest that confidence increases by working collaboratively to the point where girls are fearless and ready to attempt questions individually; the skill required for success in the final external matriculation examination.

## **Encouraging Risk-Taking in Year 12 Girls Through a Collaborative Approach to Solving Non-Routine Questions in Higher Level Mathematics**

As the Covid-19 global pandemic moves into its third year, students now sitting their final matriculation examinations have endured lockdowns over the last three years of their schooling. Lockdowns precipitated necessary innovations in learning remotely at great speed. Some schools, including Ascham School, had as little as one week to invent a plan for all students to go to remote learning. Three years on, with lessons learnt, educators have a unique opportunity to evaluate and reset the learning environment to improve on previous practices (Stannard, 2020).

The Higher School Certificate (HSC) examinations in New South Wales, Australia, are the external matriculation examinations required for entry into tertiary study. Students regard them as high stakes because they perceive that these examinations determine their future path in life. The global pandemic has exacerbated this perception in the current cohort because they have had less face-to-face teaching and examination experience due to the lockdowns.

The Mathematics subjects offered for the HSC range from non-calculus courses up to the elite Mathematics Extension 2 subject, which attracts the best mathematical minds in the state, with the ratio of boys to girls approximately 70:30. Ascham School is an independent girls' school in Sydney with a proud Mathematics tradition where the girls are encouraged to pursue the highest intellectual level within their capability. Nevertheless, girls perceive this level as the domain of boys and even highly gifted girls need to be convinced they are capable of undertaking this elite subject.

Each year, the girls sit an internal trial examination in July/August to prepare them for the final external HSC in October/November. This action research project was designed to build the confidence of the girls in the period of approximately six weeks between these two examinations and before they went on a three-week holiday/study vacation in September to prepare for the final HSC examinations.

## **Background**

In my many years of teaching matriculation Mathematics, very few students take the opportunity to do practice papers under examination conditions, correct them, and learn from their mistakes. Students will do papers, tick and cross their solutions, and copy the correct solutions, but do not actually spend time learning from them to a point where they can understand and reproduce the solution if a similar question came up again. The Year 12 cohort in this study had even less experience in preparing for a major examination because they missed their Year 11 examinations due to a 3-month lockdown during the pandemic. It was imperative, therefore, that they be given further resources and the support to build skills. This research project gave me an opportunity to compel the girls in my Mathematics Extension 2 class to utilise the weeks between the trial and the HSC examination to master those hard questions that they very often gloss over, by allowing them to work together, but without the solution.

## **Research Question**

Girls are often concerned about their relative ranking internally and performance against their peers, but the better they do as a group determines their final mark against the state. For this reason, cooperation and helping each other to improve is to the benefit of all. This led to the adoption of the collaborative strategy and research question:

Cooperation before Competition: How do shared solutions engender a fearless approach to non-routine questions in higher level Mathematics matriculation examinations?

## **Literature Review**

Statistics indicate that girls are less attracted to enrolling in higher levels of Mathematics for their matriculation examinations, with the 2021 Scaling Report from the Australian Universities' Admissions Centre (UAC) showing that girls represented 30% of candidates for the final Mathematics Extension 2 examination in the New South Wales Higher School Certificate compared with 70% boys. This is not a problem if girls just prefer to do other subjects, but it is problematic if they shy away from higher Mathematics courses because they fear the level of difficulty. In over thirty years of teaching girls

Mathematics, the latter is the most common reason quoted to me when highly capable girls do not want to undertake the higher courses of Mathematics.

Girls are more likely to enrol in more challenging Mathematics courses if they attend a girls' only school (Bridge, 2022), but while Ascham School boasts one of the highest levels of enrolment in the higher courses among girls' schools, it is still only a fraction of the boys' enrolment. If girls can overcome the apparent fear of Mathematics and be open to the challenge, therefore, they will be better prepared for the course and the final examination.

Unfortunately, research suggests that stereotypical opinions about girls', and even women's, abilities in Mathematics continue to abound. These stereotypes are generally that girls are not as good as boys at Mathematics and that boys are better suited to science careers than girls (Sumpter, et al. 2022). Further, Sumpter (2015) found that girls approached mathematical problems trying to employ algorithmic thinking that was familiar to them; that is, where they had learnt a particular method that worked for a particular type of standard or routine problem. The problem with this is that they will not know what to do if they are exposed to a non-routine question of the type that occurs in higher level Mathematics papers or the problem-solving type that occurs at the end of the paper. Thus, exposure to non-routine questions without the tools to approach them potentially leads girls to choosing other disciplines of study.

This then leads to the question of how to raise the confidence of girls to approach non-routine high-end mathematical problems with more flair or a willingness to try techniques other than algorithmic. At the same time, a recent study from the Centre for Independent Studies (2022) suggests that Mathematics teachers should return to proven methods in the Mathematics classroom. These methods include direct instruction and teaching algorithms. Furthermore, the recent popularity of inquiry-based learning appears not to have been very successful in the teaching of Mathematics (White, 2022). One could conclude from these observations, that girls need more exposure to non-routine questions, but with underlying support structures or scaffolds rather than a free rein.

Success in mathematical problem-solving is not only the result of teaching styles and classroom routines but also relies on the types of reasoning to which students are exposed or perhaps even are

innately given. Lithner (2007) proposes that if there are problems with rote learning, it is related to imitative reasoning, which is the dominant type of reasoning currently found in Mathematics Education research data. He suggests that different reasoning types can be explained when studied in relation to thinking processes and student competencies. Rote or procedural thinking is the barrier to students becoming successful problem-solvers in Mathematics and is related to difficulties in learning and achievement. The question is then, how to encourage girls in higher-order mathematical problem-solving if they have a different reasoning style from boys. Is it possible to promote a more suitable reasoning style in girls through the classroom routines?

Recent research by Mac an Bhaird (2017) suggests that giving students non-routine, unfamiliar, non-procedural, higher order, and problem-solving types of problems, such as the ones found at the end of higher-level Mathematics examinations, leads to creative thinking—the reasoning required for success in these problems. This creative reasoning is considered not gender related (Sumpter, 2015). Wirebring (2015) notes that the important protocol when exercising this process in the classroom is to not allow access to solutions; the aim being to encourage students to create their own solution, rather than enact the common teacher routine of presenting the problem and solution method which the students can then practise until mastery. According to Wirebring, this is a risky process, because the students might not be able to execute a solution, which can lead to frustration, especially when they are training for the final matriculation examination. The findings, however, were quite encouraging, where “[p]articipants who had created the solution method themselves performed better at the test questions” (Wirebring, p. 6). Further, the effects on performance and brain activity, where the cognitive load was lowered, were found to be “durable” (Wirebring, p. 6).

Wirebring’s (2015) findings are significant and have broad implications for the teaching of Mathematics in general. Christiansen et al. (2022) reinforced these findings with a sample of teenage males of varying degrees of cognitive ability. The fact that girls were not included in this study further reinforces the imperative for this action research project. If working on non-routine problems without

access to solutions enhances creativity and performance which is durable, then it is likely that girls will approach the final examination with greater confidence.

### **Research Context**

Ascham School follows the Dalton Plan, where the students are encouraged to become independent learners through the three key pillars: the lesson, the study, and the assignment. Practically, this means that two thirds of the lessons each week become lecture style and one third become tutorials where students can get individual help on their weekly set assignment, which is always due on a Tuesday morning at 8.30am. The lessons where individual help is available are called Studies.

The research took place in Term 3 over six weeks, just after the final internal trial examinations had taken place and while Year 12 were preparing for their final external matriculation examinations which give them entry to tertiary study, depending on their level of success. The cohort chosen for this research was a group of ten students who were undertaking Mathematics Extension 2, the highest level of Mathematics offered in the state of New South Wales. The stakes were high as these students usually vie for a place in elite tertiary courses, such as medicine or law. The aim of the research was to try to make the most of the girls' last six weeks of school by increasing their knowledge of, and confidence in, the coursework, in readiness for the final examination. Of course, this is always the aim when preparing for final examinations, but the interventions I decided upon would, I hoped, fulfill the aim of effecting much better targeting of the improvements that could be made in the remaining six weeks.

### **The Action**

My research explored the assertion that, in the girls' classroom, undertaking a collaborative approach to problem-solving during lessons and using non-routine problems (Mac an Bhaird, 2017) taken from recent practice papers where the solutions are not available (Wirebring, 2015), would lead to creative and durable reasoning; thus, enhancing a girl's success in the final examinations of higher Mathematics courses, irrespective of their cognitive ability (Christiansen et al., 2022).

The first survey asked the girls to analyse their performance in the paper, which topics needed more practice, how they felt, and which particular questions they found difficult or could not execute.

From the survey responses, I chose a common question listed and dedicated a lesson at the end of each week to trying to solve it. The girls were allowed to work in groups of three and share their solutions, the aim being to lower anxiety by promoting collaboration and to enhance knowledge. They were then invited to present their solution on the board to the whole group if they wished. After this, they completed the second questionnaire on how they felt about attempting the hard question.

### **Data Collection**

During the data collection period, I utilised a number of methods to ensure reliability, consistency and credibility. Much of the research involved repeated surveys taken at regular intervals to gauge trends and to identify differences, even if seemingly insignificant. Classroom observations of the collaborative exercises were taken during the project. One-to-one interviews were conducted at the end of the project and then after the final external examinations. External school data were also collected from the state published results and analysed by a consultant on behalf of the school.

In the first couple of lessons after the Trial Examinations, the students were given back their exam scripts and asked to correct their errors. They were then given a standard Mathematics Department evaluation sheet to fill in, which gauged their current position regarding mastery of topics and sense of success in relation to their performance in the recent examination. Next, as part of their weekly assignment, the girls were given a practice exam paper to complete, correct, and hand in, with a percentage mark on the front.

After each paper, I gave the girls two questionnaires to complete. The first comprised qualitative, open-ended and quantitative, Likert-style questions, while the second questionnaire focused on how they felt about attempting the hard question. This cycle ran for six weeks, after which they went on a 3-week study vacation to prepare for the final external examinations. In the last lesson, I gave them a specific timetable of practice examinations to be completed during this period.

I met four students at the beginning of Term 4 to find out to what extent the suggested preparations had been followed and if they had been successful. This feedback provided stimulus for further research (Mertler, 2020).

## **Data Analysis**

Once data across the term of the project were collected, I used a spreadsheet to input all the student responses, noting each individual's responses on each separate task. Using Mertler's (2020) method of data analysis, including "organisation, description, and interpretation," (p. 173) I examined trends over time and looked for emerging patterns by grouping the data in various categories using qualitative and quantitative tools. Individual differences as well as group data were also examined. Likert scales were implemented to measure changes over time, using averages and spread. I then analysed the observations and interviews to find any nuanced changes in attitude or behaviour towards the students' approach to their Mathematics mastery. The external school analysis was also used to see if there was any connection between the girls' confidence going into the final examination and their state performance.

## **Discussion of Findings**

Three themes became apparent as to how shared solutions to solving non-routine questions in higher levels of Mathematics lead to a confident approach in Matriculation examinations.

### **Girls Gained the Knowledge and Skills Required for Success in a Comprehensive Examination**

The first stage of this research was to gauge to what extent the girls studied for their trial examinations, the precursor to the real, final external examination for which we were preparing. On analysing their responses to the post-trial questionnaire, it became apparent that they had not really engaged with the content, nor with the depth of knowledge required to execute a lengthy examination within the set time. Almost all respondents commented that they were short of time. Student A, a high performer in the cohort, said, "Long exam - needed to work faster." Given that this cohort has borne three years of lockdowns due to Covid-19, since they were in Year 10, it is possible that they did not know how to study for a comprehensive examination because they simply had not had the opportunity before. The last set of examinations they sat at School before Year 12 would have been when they were 14 years-old in Year 9, before the pandemic.

Compelling the girls collaboratively to find solutions to difficult problems in a restricted period of time enhanced their creative power to think of unorthodox approaches and raised their confidence. They

became firmer in their knowledge, had greater accuracy, and were prepared to attempt unseen questions, believing they would achieve success.

### **The Collaborative Approach to Solutions Enabled the Girls to Become More Centred, Realistic, and Considered in Their Perceptions of Their Genuine Ability**

The intervention involved students attempting past papers, correcting them, and then re-attempting difficult questions they had in common, in collaboration with each other. Each time, they would reflect on their performance and their confidence with respect to the various questions. Each week, girls were asked how confident they felt before and after they sat the paper. In the initial weeks, depending on the perceived level of difficulty of the paper, their confidence afterwards fluctuated compared with how they felt before they sat the paper.

In Week 1, all but one respondent felt more confident after the first paper compared with before, yet in Week 2, on a scale of 1 to 10, all respondents rated their confidence afterwards as much lower. Some felt their confidence had halved. When asked how many questions they were unable to attempt and what they could do next time to improve their performance, after the first practice paper, Student B listed two questions and commented, “do another paper, write/read summaries” After the next paper, she listed twelve questions but responded with a much more specific comment about where to improve, “look at summary, review integration by parts and recurrence integrals.” This finding suggests that the girls’ sense of confidence in the early stages was directly linked to a particular experience rather than to an innate sense of their overall ability to succeed. At the same time, as they worked on challenging solutions collaboratively, they began to identify weak areas more explicitly. When asked how they felt about reproducing these difficult solutions alone, Student C responded after Week 1, “satisfying once I got it” but in Week 6, “good!”

The girls’ fluctuation in confidence continued, but each week the difference in confidence before and after decreased. In the final week, there was no difference in how they rated their confidence before and after the practice paper. This was an unexpected result and suggests that by the end of the intervention, the girls had transcended being influenced by one-off good or bad experiences and had

gained a realistic sense of their ability to cope with any style of examination. This was a very pleasing outcome, because if they could develop a fearless approach to doing practice papers under examination conditions, they were more likely to follow the holiday exam preparation timetable I supplied in readiness for the final external examination.

In the week before the final external examination, most of the girls came in to see me for final pep talks and to clarify any hazy concepts. They had all followed the examination preparation timetable I set and actually said it was helpful. After the final examination, and after the results were published, Student D came to thank me. She had raised her external mark substantially. On reflection, she noted:

That is probably why our scores (improved) kept increasing overall because since we knew we were going to be doing another practice test we had to force ourselves to understand the process used in the solutions so we could apply it again if the same technique came up. The variety of questions was huge so you knew you would see at least one of them. Having it in class helped with the switching off.

An anomalous finding, however, was the girls' belief that they would reach their potential. Girls were asked the question, "How confident are you that you can reach your potential?" On a scale of 1 to 10, Student A, a high performer in the external examination, had the lowest belief (4). Student E, however, who scored a much lower mark in the final examination, had the highest rating out of the whole class (10). More research needs to be done to find out why students believe they can or cannot reach their potential.

### **Safety in Collaboration Effected Greater Risk-Taking as Girls Increasingly Opted to Work on Their Own**

Each week, from the practice paper they had attempted individually, a nominated question that the majority had identified as being difficult to impossible, was given again. The girls were then invited to work together in groups of about three to re-attempt the question. Initially, they were very keen to work together on these difficult questions, perhaps because of safety in numbers to hide their fear, with the opportunity for weaker students to work with stronger students and avoid embarrassment. In Week 1, six

of the ten girls were initially unable to complete the given question, but after working in groups, they were all able to solve it. They were then asked if they felt confident they could reproduce it in an examination if required. Four of the ten felt they could. Student F said, “ok - just would take a lot of time, which I wouldn’t have in an exam.” However, by the last week, all but two felt they could execute the required solution. It was pleasing to observe that the stronger students were happy to lead the discussion as well and explain steps to others who were not confident about the direction to take. They were becoming more fearless. This reflects Kuhn and Villedal’s (2014) research, which found that women and girls prefer a collaborative approach when working on a project or task.

In Week 1, two thirds of the students were initially unable to complete the question by themselves, but after discussion they were all able to answer it. Further, they felt confident that they would remember the approach if a similar question arose in the examination. One student who could do the question before the discussion felt the collaboration was worthwhile because, “I found a smarter way! It was satisfying once I got it.” A volunteer then gave the solution on the board and the group gave an appraisal, looking for efficiencies. Students also rated the level of difficulty of the question on a scale from 1 to 10.

In Week 1, the difficulty ratings ranged from 7 to 9. As the weeks went by, I tried to select increasingly difficult questions for the girls to attempt. It was interesting to observe two things. The first was that the girls’ perception of the level of difficulty of the questions actually diverged, now ranging from 3 to 10. Again, this suggests a more realistic confidence in their ability to attempt a hard exam. The second was that they gradually weaned themselves off the group effort and began working by themselves on these challenging questions. In the final week, Student C, in response to the question, “are you now able to do this question after discussion?” said, “[I] didn’t really discuss.” This was a desirable outcome as eventually the students needed a fearless approach to sit the external examination as an individual effort. No room for group work there.

## Conclusion

Trying to make use of the time left at school between trial examinations and the final external examination to build knowledge and confidence, especially with the higher level Mathematics examinations, has always been a challenge; one which was exacerbated by the pandemic. Students will often do practice papers and correct them, copying solutions they did not get correct, but they then move on and do not ingest the method or approach. Consequently, if they encounter the same question again, they may get it wrong again.

The purpose of my intervention was to focus on those difficult questions that most students have no idea about and to compel the girls to re-attempt these questions in a safe environment, with peer and teacher support. The feedback that they then felt ready to attempt the practice papers on their own in readiness for the final external examination, coupled with their actual success in it as evidenced by the published results, reflects the success of this project in raising the girls' confidence and engendering a fearless approach to mathematical problem-solving.

Some anomalies or unexpected responses provided aspects for further research, such as why girls feel uncertain about their ability in Mathematics, even though they are clearly strong students compared with the rest of the state. The success of collaboratively developing solutions to difficult non-routine problems lies in the fact that the students believed that they could now do these challenging Mathematics questions and get them correct, which augured well for the final examination. Some final comments from the students suggest the positive impact of this action research: "I have really appreciated all your energy, joy, and HSC practice paper schedules; they have all helped me get through the most difficult HSC course," and, "I've enjoyed learning from you and also being your guinea pig. In case you didn't know, Maths was my favourite subject."

## Reflection

I applied for the GARC program because for decades I have wanted to learn the formal process and skills required for producing an academic paper worthy of publication. Andrew Powell, Head of School, has been most supportive and Matt Massey, Director of Professional Learning, and his team facilitated the finance to attend the conferences. Joanne Manning, Liz McCabe, Gabe Bonney, Paige Macdonnell and their teams have been most helpful with the logistics of publications. This has been a stimulating experience for me, and I now appreciate the effort it takes to produce one such paper. Had I known what was involved and the number of weeks required, I might have chosen to write the surveys earlier or refine them, as my original questions changed, and I wanted to research other aspects. I feel very strongly that girls should do Mathematics and researching the papers that already exist on this topic has been a marvellous experience. I would highly recommend this program to any budding researcher who cares about their teaching practice.

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